

MATH 2D Prep: Local Extreme Values

1. Find all critical numbers of $f(x) = x^3$. Does $f(x)$ have any local extreme value? Why?

Solution:

$$f'(x) = 3x^2 = 0 \Rightarrow x = 0$$

So $f(x)$ has a unique critical number $x = 0$.

$f'(x) = 3x^2 > 0$ on both sides of $x = 0$, so by First Derivative Test, $f(x)$ has no local extreme value at $x = 0$. $f(x)$ cannot have local extreme value anywhere else, because $x = 0$ is the only critical number. So $f(x)$ does not have any local extreme value.

Note: Second Derivative Test gives no conclusion in this case, so it's useless in this problem.

2. Find all local extreme values of $g(x) = -x^4 + 2x^2$

Solution:

$$g'(x) = -4x^3 + 4x = -4x(x^2 - 1) = -4x(x - 1)(x + 1)$$

$$g'(x) = 0 \Rightarrow x = 0 \text{ or } 1 \text{ or } -1$$

So critical numbers are 0, 1 and -1 .

$$g''(x) = -12x^2 + 4, \quad g''(1) = g''(-1) = -8 < 0, \quad g''(0) = 4 > 0$$

So $g(x)$ has local maximum at 1 and -1 , has local minimum at 0.

Local maximum value is $g(1) = g(-1) = 1$ at $x = 1$ and $x = -1$,

local minimum value is $g(0) = 0$ at $x = 0$.